# nag_arctanh (s11aac)

## 1. Purpose

**nag_arctanh (s11aac)** returns the value of the inverse hyperbolic tangent, $\operatorname{arctanh} x$.

## 2. Specification

```
#include <nag.h>
#include <nags.h>

double nag_arctanh(double x, NagError *fail)
```

## 3. Description

The function calculates an approximate value for the inverse hyperbolic tangent of its argument, $\operatorname{arctanh} x$.

For $x^2 \leq \frac{1}{2}$ the function is based on a Chebyshev expansion.

For $\frac{1}{2} < x^2 < 1$,

$$\operatorname{arctanh} x = \tfrac{1}{2} \ln \left( \frac{1+x}{1-x} \right).$$

## 4. Parameters

**x**

Input: the argument $x$ of the function.
Constraint: $|\mathbf{x}| < 1.0$.

**fail**

The NAG error parameter, see the Essential Introduction to the NAG C Library.

## 5. Error Indications and Warnings

**NE_REAL_ARG_GE**

On entry, $|\mathbf{x}|$ must not be greater than or equal to 1.0: $\mathbf{x} = \langle value \rangle$.
The function has been called with an argument greater than or equal to 1.0 in magnitude, for which arctanh is not defined. The result is returned as zero.

## 6. Further Comments

### 6.1. Accuracy

If $\delta$ and $\epsilon$ are the relative errors in the argument and result, respectively, then in principle

$$|\epsilon| \simeq \left| \frac{x}{(1-x^2)\operatorname{arctanh} x}\, \delta \right|.$$

That is, the relative error in the argument, $x$, is amplified by at least a factor

$$\frac{x}{(1-x^2)\operatorname{arctanh} x}$$

in the result. The equality should hold if $\delta$ is greater than the ***machine precision*** ($\delta$ due to data errors etc.), but if $\delta$ is simply due to round-off in the machine representation then it is possible that an extra figure may be lost in internal calculation round-off.

The factor is not significantly greater than one except for arguments close to $|x| = 1$. However, in the region where $|x|$ is close to one, $1 - |x| \sim \delta$, the above analysis is inapplicable since $x$ is bounded by definition, $|x| < 1$. In this region where arctanh is tending to infinity we have

$$\epsilon \sim 1/\ln \delta$$

which implies an obvious, unavoidable serious loss of accuracy near $|x| \sim 1$, e.g. if $x$ and 1 agree to 6 significant figures, the result for $\operatorname{arctanh} x$ would be correct to at most about one figure.

### 6.2. References

Abramowitz M and Stegun I A (1968) *Handbook of Mathematical Functions* Dover Publications, New York ch 4.6 p 86.

## 7. See Also

None.

## 8. Example

The following program reads values of the argument $x$ from a file, evaluates the function at each value of $x$ and prints the results.

### 8.1. Program Text

```
/* nag_arctanh(s11aac) Example Program
 *
 * Copyright 1989 Numerical Algorithms Group.
 *
 * Mark 2 revised, 1992.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

main()
{
  double  x, y;

  Vprintf("s11aac Example Program Results\n");
  Vscanf("%*[^\n]");    /* skip the first input line */
  Vprintf("     x           y\n");
  while (scanf("%lf", &x) != EOF)
    {
      y = s11aac(x, NAGERR_DEFAULT);
      Vprintf("%12.3e%12.3e\n", x, y);
    }
  exit(EXIT_SUCCESS);
}
```

### 8.2. Program Data

```
s11aac Example Program Data
      -0.5
       0.0
       0.5
      -0.9999
```

### 8.3. Program Results

```
s11aac Example Program Results
     x           y
  -5.000e-01  -5.493e-01
   0.000e+00   0.000e+00
   5.000e-01   5.493e-01
  -9.999e-01  -4.952e+00
```